

# The spammed Code Offset Method

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[eprint.iacr.org/2013/527](http://eprint.iacr.org/2013/527)

# Outline

- Helper data schemes
  - privacy-preserving biometric databases
  - Physically Obfuscated Keys
- The Code Offset Method
- Adding fake enrollment data
  - while retaining efficient reconstruction
  - LDPC codes, syndromes, ...
- Analysis
  - security
  - storage/work

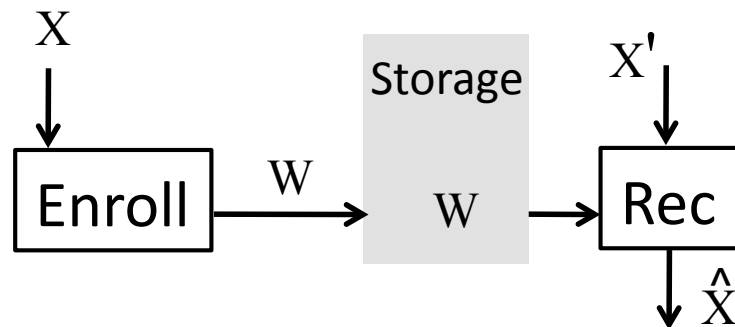
} trade-off

# Scenario 1: privacy-preserving biometrics DB

Aim: store only the hash of a user's fingerprint/iris/...

Problem: **noise**

Solution: helper data scheme (Secure Sketch)



Desired properties:

- High prob. of correct reconstruction.
- $W$  does not leak much about  $X$ .

*Database entry:*



Figure of merit:  $H(X|W)$

# Scenario 2: Physically Obfuscated Key

Aim: Alternative technology for read-proof key storage.

Obtain key from measurement on complex physical system ("PUF").

Problem: **noise**

Solution: helper data scheme

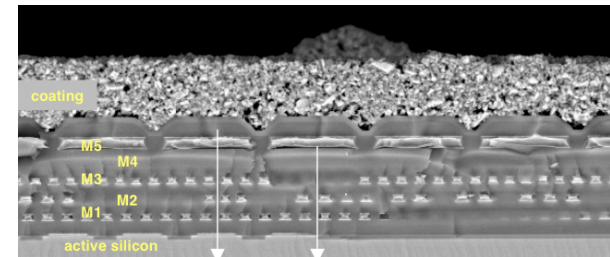
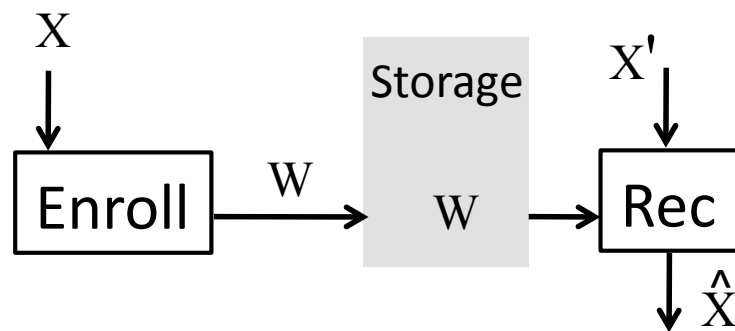
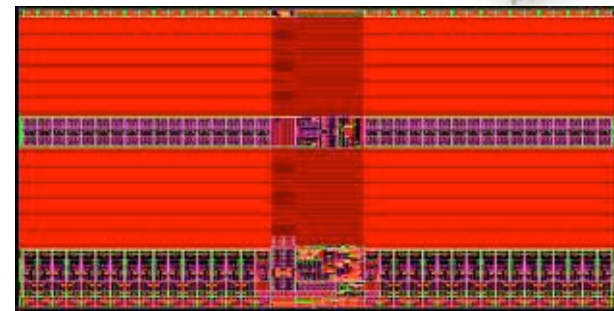


Figure of merit:  $H_2(X|W)$



# Intermezzo: Error-correcting codes

k-bit message  $\mu$ .

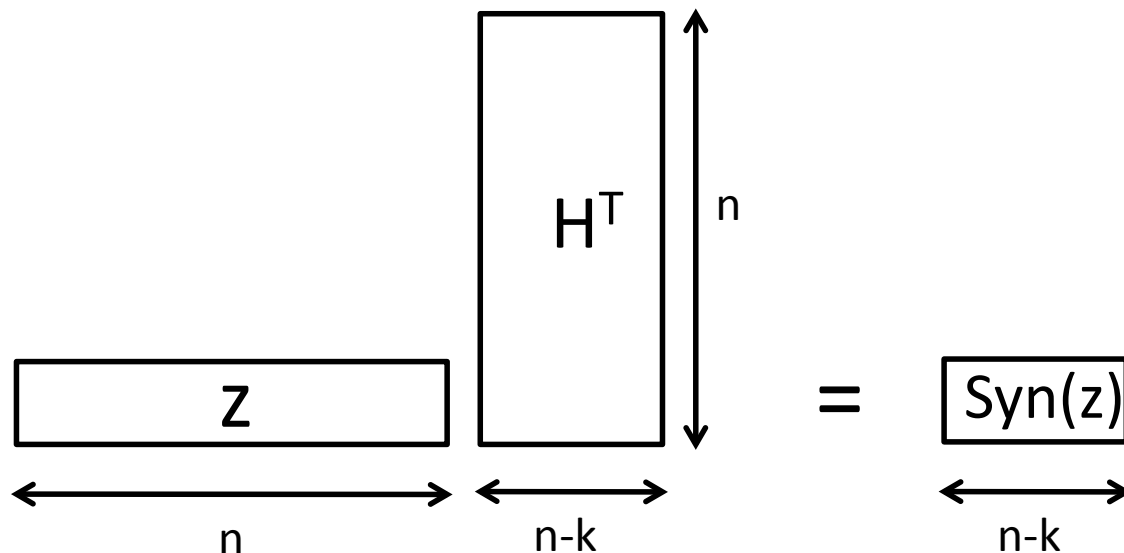
n-bit codeword  $C_\mu$ .

n-bit noise pattern  $e$ .

$z = C_\mu + e$

Syndrome  $\text{Syn}(z) = \text{Syn}(C_\mu) + \text{Syn}(e) = \text{Syn}(e)$

$$C_\mu H^T = \underline{0}$$



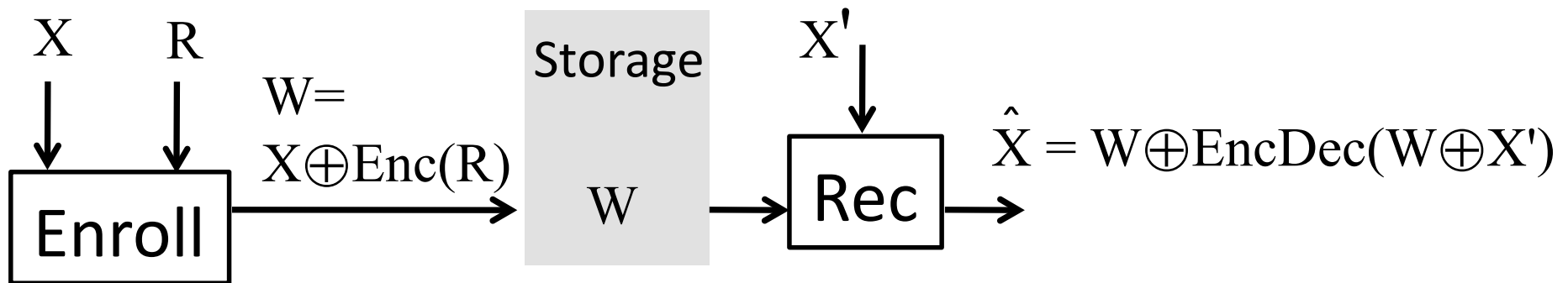
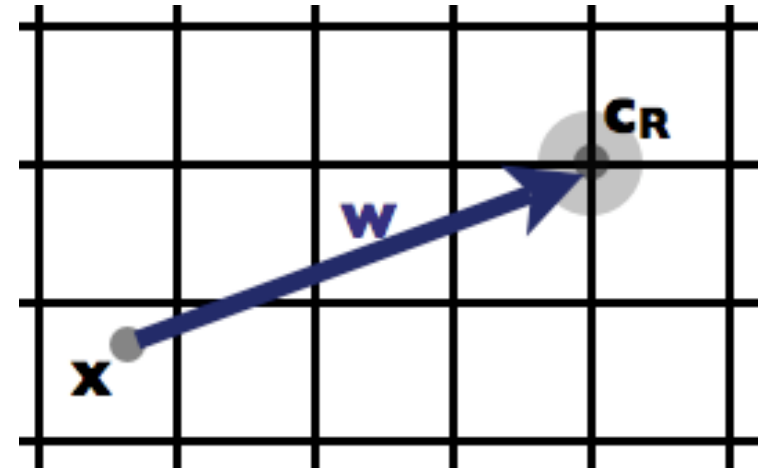
"Low-Density Parity Check" matrix [Gallager 1960]

# Code Offset Method

[Juels & Wattenberg 1999]

## "The mother of all Secure Sketches"

- Source  $X \in \{0,1\}^n$ .
- Uniformly random  $R \in \{0,1\}^k$ .
- Binary linear error correcting code (Enc, Dec).  
Message size  $k$ ; codeword size  $n$ .

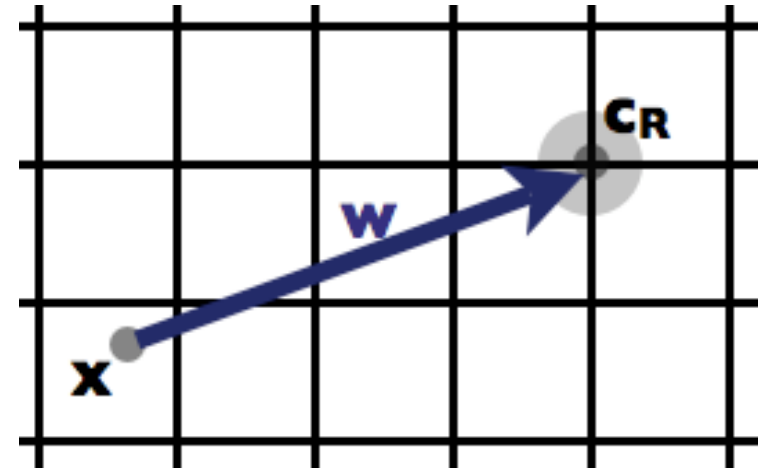


# Code Offset Method: analysis

## How good is this?

If  $X$  is uniform:

- $H(R|W)=H(R)$ ; **no leakage about  $R$ !**
- $H(X|W) = H(R) = k$   
     $W$  leaks  $n-k$  bits about  $X$



## If $X$ is *not* uniform

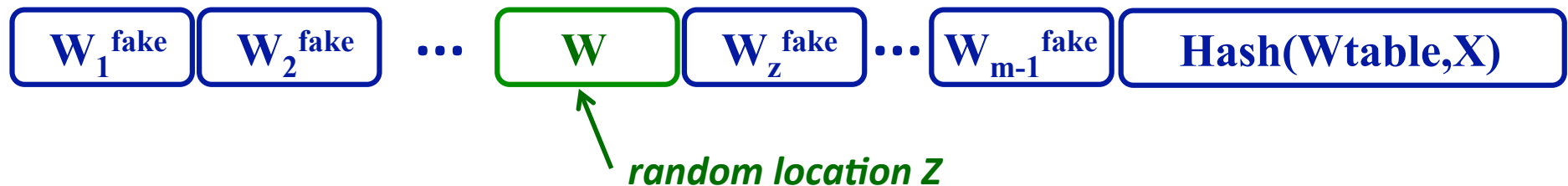
- $W$  leaks about  $R$
- $W$  still reveals  $\text{Syn}(X)$

**Can we do better?**

# Fake helper data

Idea: hide  $W$  in lots of fake helper data (with same distribution)

Biometrics database, entry for one user:



Legitimate party:

- Has  $X'$
- Reconstruction by brute force: Try all entries

Attacker:

- Does not have  $X'$
- Brute force attack
- effort multiplied by  $m/2 \Rightarrow \log(m/2)$  bits of security gained



# More efficient scheme

- Use LDPC code
  - parity check matrix is sparse
  - $X' \approx X$  implies  $\text{Syn}(X') \approx \text{Syn}(X)$
- Store  $\text{Syn}(X) = \text{Syn}(W)$  and all  $\text{Syn}(W^{\text{fake}})$ 
  - can be computed from  $W$  and  $W^{\text{fake}}$
  - reveals nothing new
  - Code Offset Method possible with only syndrome



- Fast reconstruction:**
- Compute  $\text{Syn}(X')$
  - Prioritize entries with  $\text{Syn}(W_i) \approx \text{Syn}(X)$ .

# Security analysis

Without spam:  $H(X|W) = H(\text{Syn } X)$

With spam:  $H(X|\Omega) \geq H(X|W) + \log m - \frac{m-1}{\ln 2} \mathbb{E}_x q_{\text{Syn}(x)}$

$$H(X|\Omega) \geq H(X) - \frac{1}{m} \cdot \frac{2^{n-k} - 1}{\ln 2}$$

$\Omega$ : the helper data list

$$q_a = \text{Prob}[\text{Syn } X=a]$$

Typically,  $\frac{m-1}{\ln 2} \mathbb{E}_x q_{\text{Syn}(x)}$  is of order  $\frac{m}{2^{n-k}}$

$m \rightarrow 2^{n-k}$ : Leakage gets close to zero.

# The size of the table (assuming LDPC)

*biometrics (1 user)*

*phys. obfuscated key*

		k=64			k=128	
#err	n	log m	Mem	n	log m	Mem
1	72	4	<b>16 B</b>	138	5	<b>40 B</b>
		8	<b>0.3 KB</b>		10	<b>1.2 KB</b>
2	78	7	<b>0.2 KB</b>	146	9	<b>1.1 KB</b>
		14	<b>29 KB</b>		18	<b>0.6 MB</b>
3	85	10.5	<b>3.8 KB</b>	154	13	<b>26 KB</b>
		21	<b>5.5 MB</b>		26	<b>0.2 GB</b>

- *n values are approximate*
- *Listed values for log m:  $(n-k)/2$  and  $n-k$*
- *Choose m that fits in memory  $\Rightarrow$  sec. gain  $\log(m)-1$  bits*

# Summary

We added a new "knob" to the Code Offset Method

- better use of source entropy
  - price: size of enrollment data
- } *trade-off*
- security analysis: Shannon entropy
  - Rényi entropy [not shown]
  - interesting for low source entropy

## Work in progress:

- explicitly choose LDPC codes
- *generate* the table (with PRNG)
  - security  $\leftrightarrow$  memory tradeoff becomes security  $\leftrightarrow$  work tradeoff



## Acknowledgements

- Ruud Pellikaan
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